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# STUDY ON THE SETTLEMENT OF TUNNEL BOTTOM AND PRESSURE OF ROCK MASS BASED ON CURVED BEAM ON ELASTIC FOUNDATION THEORY

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## ABSTRACT

Tunnel invert is a weak section of curved beam on tunnel foundation, and it is easy to break down. Based on the curved beam theory on elastic foundation, the curved beam model of tunnel invert was established, the displacement equation of tunnel invert under external load was deduced, and the formula of settlement of tunnel bottom and the pressure of rock mass was presented. By means of the calculating formula, the distribution law of settlement of tunnel bottom and pressure of rock mass were obtained when tunnel bottom was strengthened and not strengthened by high pressure jet grouting pile. The final formula in the paper is precise to predict the settlement of tunnel bottom and pressure of rock mass, so it is of great value for tunnel design and construction.

## KEYWORDS

Tunnel invert, Curved beam, Elastic foundation, Displacement equation, Pressure of rock mass

## INTRODUCTION

There are many problems in the construction of curved beams, such as embankment culvert, underground pipeline and tunnel lining, and the curved beam model can be used to solve them [1-3]. At present, the research of curved beams is mostly concentrated on the study of the basic theory [4-5]. Pagano et al. [6-7], Chien et al. [8], Shen et al. [9], and Tornabene et al. [10] used the linear elastic theory to study laminated composite plates under the condition of column bending and gave the exact solutions. Lekhnitskii et al. [11] gave the general solution of anisotropic bending beam under moment load. This method was also adopted to calculate the interlaminar tensile stress for four point bending experiment of curved beam (ASTMD6415/D6415M-06a) [12]. Shenoi et al. [13], Arici et al. [14] established the model for bending behaviour of elastically curved beams and obtained the elastic solution of the anisotropic beam. Higher-order shear deformation theory can accurately calculate in-plane deformation and stress for shell structure with span to thickness ratio than four, but cannot calculate the interlaminar stresses for composite plates and shells [15, 16]. Aköz et al. [17] used the finite element method to analyse circular beam with variable cross section on elastic foundation under arbitrary load. Banan et al. [18] found the general finite element formula for spatial curved beams and arches on elastic foundation. Çalım et al. [19, 20] presented an effective method for analysing the dynamic performance of straight beams and curved beams on elastic foundation. Wang et al. [21], Öz et al. [22], Chen et al. [23], Malekzadeh et al. [24], and Wei et al. [25] studied the vibration problem of curved beam on elastic foundation. Based on Euler beam theory, Zhong et al. [26], Adineh et al. [27], Cong et al. [28], Duc et al. [29, 30], and Dung et al. [31] studied the nonlinear transient thermal response for FGM beam with unstable heat conduction and revealed the mechanical characteristics of FGM beams on the basis of tension free. The academic research on the curved beam on elastic foundation has made a wealth of research results, but there is little research in practical engineering applications.

In tunnelling engineering, the tunnel lining is closely contacted with rock mass, the tunnel lining can be regarded as curved beam on elastic foundation. Dai et al. [32] put forward the calculation model of tunnel lining deformation and internal force on the basis of the Qingdao overlapping tunnel on the beam theory on elastic foundation. Based on principle of initial parameter method, Sun et al. [33] established the initial parameter matrix equation for solving the internal force and deformation of concrete lining on curved beam on elastic foundation. Through on-site monitoring of steel arch stress and pressure of rock mass, Wen et al. [34] deduced an analytical formula for internal force of the primary support of tunnel on curved beam on elastic foundation. Based on the beam model on double elastic foundation and influence of uneven settlement of soft soil, Li et al. [35] established a plane numerical model and analysed the arrange of settlement joint and the longitudinal stress and deformation of the tunnel. In summary, more emphasis focuses on the analysis of the internal force and the uneven settlement in the longitudinal range of the superstructure of the tunnel, while the analysis of the tunnel invert is relatively rare.

In this paper, the curved beam model of the tunnel invert was established and the displacement equation of curved beam on elastic foundation under external load was deduced based on the differential equation of beam on elastic foundation. The displacement equation was used to calculate the settlement of tunnel bottom and the pressure of rock mass when the tunnel foundation of soft loess was strengthened and not strengthened by high pressure jet grouting pile.

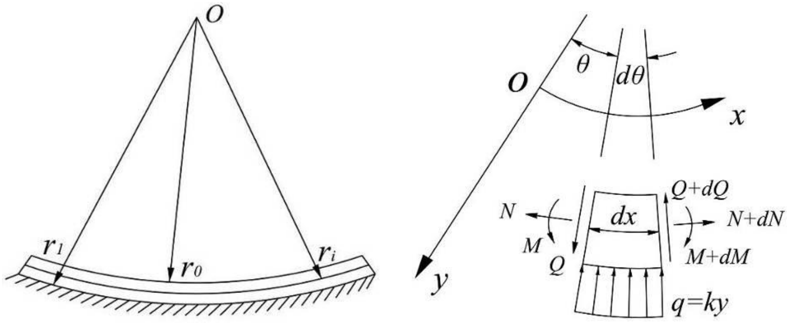
## DIFFERENTIAL EQUATION OF CURVED BEAM ON ELASTIC FOUNDATION

The Winkler Foundation assumes that the settlement at any point on the surface of the foundation is proportional to the pressure on the unit area of the point. This assumption is the foundation for series of independent spring simulation on rigid base. When a point on the foundation surface is subjected to pressure $p$, because the springs are independent of each other, only the local settlement $y$ is generated at this point, and no settlement occurs in other places. The foundation model is also called the local elastic foundation model. The formula is expressed in Equation 1.

$$p=Ky \tag{1}$$

Where $K$ is the coefficient of foundation reaction which represents the pressure intensity required to produce the unit deformation at a point on the foundation ($kN/m^3$); p is the pressure intensity at any point on the foundation ($kPa$) and $y$ is the deformation of the foundation at the point of pressure ($m$).

The local elastic foundation model is still applicable in the derivation of curved beam on elastic foundation. There is a constant cross-section beam on the Winkler Foundation (Figure 1(a)). The inner and outer radius of curved beam is $r_0$ and $r_1$ respectively, and the thickness of curved beam is $h=r_1-r_0$, the sectional area is $A$, the sectional inertia moment is $I$, and the centre radius is $ri=(r_1+r_0)/2$. Assuming that there is no friction effect, the resistance of the foundation is proportional to the radial displacement of the curved beam.




*(a) Curved beam with constant cross section (b) The force of element*

*Fig. 1 - Schematic diagram of curved beam on elastic foundation*

We selected an arbitrary element $r_i d\theta$ from the beam as the object of analysis, as shown in Figure 1(b). $y(\theta)$ is the radial displacement; $K$ is the coefficient of foundation reaction; $M(\theta)$ is the bending moment of initial section; $Q(\theta)$ is the shear force and $N(\theta)$ is the axial force. All the forces are positive in the direction of the Figure 1.

According to the force state of the element, as shown in Figure 1(b), the radial, tangential and centre moment balance equations of the infinitesimal element are established, as shown in Equations 2 to 4.

$$N(\theta)d\theta + Ky(\theta)r_1 d\theta + dQ(\theta) = 0 \tag{2}$$

$$Q(\theta)d\theta - dN(\theta) = 0 \tag{3}$$

$$dM(\theta) - Qr_i d\theta = 0 \tag{4}$$

The radial displacement $y(\theta)$ of the curved beam has following relationship with internal force [34]:

$$\frac{d^2y(\theta)}{d\theta^2} + y(\theta) = \frac{M(\theta)r_i^2}{EI} + \frac{N(\theta)r_i}{EA} \tag{5}$$

Where $E$ is the elastic modulus of th curved beam.

To derive $\theta$ in the Equation 5 once and three times, we can get the following equation:

$$\frac{d^3y(\theta)}{d\theta^3} + \frac{dy(\theta)}{d\theta} = \frac{r_i^2 dM(\theta)}{EId\theta} + \frac{r_i dN(\theta)}{EAd\theta} \tag{6}$$

$$\frac{d^5y(\theta)}{d\theta^5} + \frac{d^3y(\theta)}{d\theta^3} = \frac{r_i^2 d^3M(\theta)}{EId\theta^3} + \frac{r_i d^3N(\theta)}{EAd\theta^3} \tag{7}$$

Then we can obtain the Equation 8 by adding equation 6 and 7:

$$\frac{d^5y(\theta)}{d\theta^5} + 2\frac{d^3y(\theta)}{d\theta^3} + \frac{dy(\theta)}{d\theta} = \frac{r_i^2 d^3M(\theta)}{EId\theta^3} + \frac{r_i d^3N(\theta)}{EAd\theta^3} + \frac{r_i^2 dM(\theta)}{EId\theta} + \frac{r_i dN(\theta)}{EAd\theta} \tag{8}$$

We can get a five order radial displacement $y(\theta)$ differential equation with constant coefficients

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based on simultaneous Equation 2, 4 and 8:

$$\frac{d^5 y(\theta)}{d\theta^5} + 2\frac{d^3 y(\theta)}{d\theta^3} + n^2\frac{dy(\theta)}{d\theta} = 0 \tag{9}$$

where: $n^2 = 1 + Kr_1(\frac{r_i^3}{EI} + \frac{r_i}{EA})$

The characteristic equation of the general solution of Equation 9 is as follows:

$$\lambda^5 + 2\lambda^3 + n^2\lambda = 0$$

The solution of the characteristic equation is:

$$\lambda_1 = 0, \lambda_{2,3} = \alpha \pm i\beta, \lambda_{4,5} = -(\alpha \pm i\beta)$$

where: $\alpha = \sqrt{\frac{n-1}{2}}$; $\beta = \sqrt{\frac{n+1}{2}}$; $i = \sqrt{-1}$.

So the general solution of the Equation 9 is:

$$y(\theta) = C_0 + C_1 ch\alpha\theta cos\beta\theta + C_2 sh\alpha\theta cos\beta\theta + C_3 ch\alpha\theta sin\beta\theta + C_4 sh\alpha\theta sin\beta\theta \tag{10}$$

The Equation 11 can be obtained by simultaneous Equation 3, 4 and 6:

$$Q(\theta) = (\frac{r_i^3}{EI} + \frac{r_i}{EA})^{-1}[\frac{d^3 y(\theta)}{d\theta^3} + \frac{dy(\theta)}{d\theta}] \tag{11}$$

The Equation 12 can be got by simultaneous Equation 2 and 11:

$$N(\theta) = -(\frac{r_i^3}{EI} + \frac{r_i}{EA})^{-1}[\frac{d^4 y(\theta)}{d\theta^4} + \frac{d^2 y(\theta)}{d\theta^2}] - Kr_1 y(\theta) \tag{12}$$

And the Equation 13 can be obtained by simultaneous Equation 5 and 12:

$$M(\theta) = \frac{EI}{r_i^2}\left\{\frac{d^2 y(\theta)}{d\theta^2} + y(\theta) + (\frac{r_i^2 A}{I} + 1)^{-1}[\frac{d^4 y(\theta)}{d\theta^4} + \frac{d^2 y(\theta)}{d\theta^2}] + \frac{r_i}{EA}Kr_1 y(\theta)\right\} \tag{13}$$

Finally, we can get analytical equation of internal force of curved beam by simultaneous Equation 2 to 6 and Equation 10:

$$Q(\theta) = T[C_1(A_1 sh\alpha\theta cos\beta\theta + A_2 ch\alpha\theta sin\beta\theta) + C_2(A_1 ch\alpha\theta cos\beta\theta + A_2 sh\alpha\theta sin\beta\theta) + C_3(A_1 sh\alpha\theta sin\beta\theta - A_2 ch\alpha\theta cos\beta\theta) + C_4(A_1 ch\alpha\theta sin\beta\theta - A_2 sh\alpha\theta cos\beta\theta)] \tag{14}$$

$$N(\theta) = -Kr_1 C_0 - C_1(2\alpha\beta T sh\alpha\theta sin\beta\theta) - C_2(2\alpha\beta T ch\alpha\theta sin\beta\theta) + C_3(2\alpha\beta T sh\alpha\theta cos\beta\theta) + C_4(2\alpha\beta T ch\alpha\theta cos\beta\theta) \tag{15}$$

$$M(\theta) = [(Kr_1 B_1 + 1)C_0 + C_1(2\alpha\beta)(B_1 T - 1)sh\alpha\theta sin\beta\theta + C_2(2\alpha\beta)(B_1 T - 1)ch\alpha\theta sin\beta\theta - C_3(2\alpha\beta)(B_1 T - 1)sh\alpha\theta cos\beta\theta - C_4(2\alpha\beta)(B_1 T - 1)ch\alpha\theta cos\beta\theta] / B_2 \tag{16}$$

Where: $A_1 = -2\alpha\beta^2$; $A_2 = -2\alpha^2\beta$; $B_1 = r_i/(EA)$; $B_2 = r_i^2/(EI)$; $T = (\frac{r_i^3}{EI} + \frac{r_i}{EA})^{-1}$

## THE DIFFERENTIAL EQUATION OF CURVED BEAM ON ELASTIC FOUNDATION UNDER SYMMETRIC LOAD

Section forms of the highway tunnel are generally symmetrical about the vertical axis, and no asymmetric loading tunnel, so it can be assumed that the load of tunnel rock mass is also symmetrical about vertical axis. The section where the tunnel centre is located is taken as the initial section, the symmetry of structure and load can be used.

$$[\frac{dy(\theta)}{d(\theta)}]_{\theta=0} = 0 \,,$$

$$[Q(\theta)]_{\theta=0} = 0$$

That is:

$$\alpha C_2 + \beta C_3 = 0$$

$$\beta C_2 - \alpha C_3 = 0$$

So we can get $C_2 = C_3 = 0$.

Then, Equations 10 and 14 to 16 can be simplified as follows:

$$y(\theta) = C_0 + C_1 ch\alpha\theta cos\beta\theta + C_4 sh\alpha\theta sin\beta\theta \tag{17}$$

$$-Q_0 = T[C_1(A_1 sh\alpha\theta_0 cos\beta\theta_0 + A_2 ch\alpha\theta_0 sin\beta\theta_0) + C_4(A_1 ch\alpha\theta_0 sin\beta\theta_0 - A_2 sh\alpha\theta_0 cos\beta\theta_0)] \tag{18}$$

$$-Q_0 = T[C_1(A_1 sh\alpha\theta_0 cos\beta\theta_0 + A_2 ch\alpha\theta_0 sin\beta\theta_0) + C_4(A_1 ch\alpha\theta_0 sin\beta\theta_0 - A_2 sh\alpha\theta_0 cos\beta\theta_0)] \tag{19}$$

$$-Q_0 = T[C_1(A_1 sh\alpha\theta_0 cos\beta\theta_0 + A_2 ch\alpha\theta_0 sin\beta\theta_0) + C_4(A_1 ch\alpha\theta_0 sin\beta\theta_0 - A_2 sh\alpha\theta_0 cos\beta\theta_0)] \tag{20}$$

In Equations 17 - 20, the internal force analytical equation of curved beam on elastic foundation can be obtained after the integral constants $C_0$, $C_1$ and $C_4$ are calculated. And the integral constants $C_0$, $C_1$ and $C_4$ can be obtained by the boundary conditions at both ends of the curved beam.

## ESTABLISHMENT OF CURVED BEAM MODEL ON ELASTIC FOUNDATION OF TUNNEL INVERT

When the tunnel invert was completed, it would be affected by the weight of the lining and the load of the rock mass. The settlement of tunnel bottom would produce pressure on the foundation; similarly, the tunnel foundation would also produce the counterforce to the tunnel invert. Therefore, the tunnel invert could be regarded as a curved beam on the elastic foundation. The self-weight of tunnel lining and the pressure of rock mass could be equivalent to the load acting on both ends of tunnel invert, and the boundary conditions at the ends of the tunnel invert could be obtained by the equivalent load. Assuming that the tunnel invert was rigidly connected with the secondary lining of the tunnel, the equivalent load could be simplified as shear $Q_0$, axial force $N_0$ and bending moment $M_0$, as shown in Figure 2.

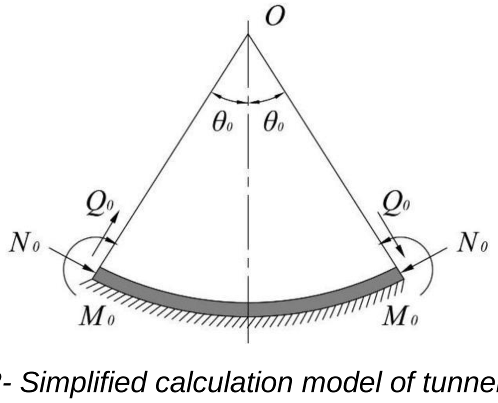


*Fig. 2- Simplified calculation model of tunnel invert*

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Then the boundary condition of the beam end was:

$$Q(\theta_0) = -Q_0$$
$$N(\theta_0) = -N_0 \tag{21}$$
$$M(\theta_0) = -M_0$$

So we could establish a three-variable linear equation by simultaneous Equations 18 - 20, 21, as shown in Equation 22 - 24:

$$-Q_0 = T[C_1(A_1 sh\alpha\theta_0 cos\beta\theta_0 + A_2 ch\alpha\theta_0 sin\beta\theta_0) + C_4(A_1 ch\alpha\theta_0 sin\beta\theta_0 - A_2 sh\alpha\theta_0 cos\beta\theta_0)] \tag{22}$$

$$-N_0 = -Kr_1C_0 - C_1(2\alpha\beta T sh\alpha\theta_0 sin\beta\theta_0) + C_4(2\alpha\beta T ch\alpha\theta_0 cos\beta\theta_0) \tag{23}$$

$$-M_0 = [(Kr_1B_1+1)C_0 + C_1(2\alpha\beta)(B_1T-1)sh\alpha\theta_0 sin\beta\theta_0 - C_4(2\alpha\beta)(B_1T-1)ch\alpha\theta_0 cos\beta\theta_0]/B_2 \tag{24}$$

The pressure of rock mass of tunnel invert could be calculated according to literature (JTGD70-2004). The internal force of the corner of tunnel invert could be obtained according to the elastic centre method in Structural Mechanics. Then the shear force $Q_0$, axial force $N_0$ and bending moment $M_0$ acting on both ends of tunnel invert could be obtained.

## CALCULATION EXAMPLES OF THE SETTLEMENT OF TUNNEL BOTTOM AND PRESSURE OF ROCK MASS IN THE DAYOUSHAN TUNNEL

The foundation reinforcement section of the Dayoushan tunnel was selected as the typical section, and the above theoretical formula was tested by the engineering case. Firstly, the settlement of the tunnel bottom and pressure of rock mass were calculated when the foundation was not strengthened, and the law of the force and settlement of the tunnel bottom was explored. Then the settlement and pressure of rock mass of foundation reinforcement by high pressure jet grouting pile were calculated.

The composite lining was adopted in the Dayoushan tunnel, the excavation radius was 6.41m, and the excavation was constructed by bench method. The primary support of shallow section was sprayed with C25 concrete, and its thickness was 26cm; the secondary lining used C25 reinforced concrete, and its thickness was 50cm. The analysis of unit length of tunnel invert in shallow section was carried out. The geometric parameters and calculation parameters of tunnel invert were shown in Table 1 and Table 2 respectively.

*Tab.1 - Geometric parameters of tunnel invert*

| Thickness $h$ （m） | External radius $r_1$ （m） | Inner radius $r_0$（m） | Central angle $\theta_0$ （°） | Cross-sectional area $A$ （$m^2$） |
|---|---|---|---|---|
| 50 | 12.561 | 12.061 | 26.9 | 5.87 |

*Tab.2 - Calculation parameters of tunnel invert*

| Parameters | Value | Material |
|---|---|---|
| Elastic modulus of concrete $E_c$（GPa） | 29.5 | C25 concrete |
| Elastic modulus of reinforcement $Es$（GPa） | 200 | Φ8 double layer steel mesh |
| Sectional area of tunnel invert steel bar（$m^2$） | 5.03×10$^{-5}$ | Φ8 double layer steel mesh |
| Coefficient of foundation reaction $K_1$（$kN/m^3$） | 8000 | Soft foundation |
| Coefficient of foundation reaction $K_2$（$kN/m^3$） | 60000 | Foundation reinforcement by high pressure jet grouting pile |

The composite elastic modulus of reinforced concrete could be calculated according to formula (25):

$$EA=E_cA_c+E_sA_s \tag{25}$$

Where $E_c$ and $E_s$ is the elastic modulus of concrete and steel bars of curved beam on foundation reinforcement, respectively; $A_c$ and $A_s$ is the sectional area of concrete and steel bars.

$$E=(E_cA_c+E_sA_s)/A=29.54\ (GPa)$$

The section ZK3+160 of Dayoushan tunnel was selected for the study. The depth of this section was 30m, and the rock mass was 13m collapsible loess and 17m non collapsible loess. The lateral pressure coefficient was calculated according to the lateral pressure and vertical pressure obtained from the previous monitoring, and $\lambda=0.5$. The pressure of rock mass of shallow tunnel was calculated according to the literature (JTGD70-2004), the vertical pressure $q=348kN/m$ and horizontal lateral pressure $e=174\ kN/m$ were calculated, after the load was reduced by 60% (The pressure of rock mass acting on secondary lining accounted for about 60%), so $q_1=208kN/m$ and $e_1=104kN/m$. The internal force at the corner of tunnel invert could be obtained according to the elastic centre method in Structural Mechanics. The shear force $Q_1=342kN$, the axial force $N_1=1233kN$ and the bending moment $M_0=591\ kN\cdot m$ were obtained.

Considering the influence of the weight of secondary lining, the cross section area of secondary lining was $11.52m^2$, and the bulk density was $25kN/m^3$. So the weight of unit length of the secondary lining was $281.25kN$. The force produced by the weight of secondary lining acting on the tunnel invert could be calculated by the following formula:

Shear force: $Q_2=(281.25/2)\times\cos\theta_0=125\ (kN)$

Axial force: $N_2=(281.25/2)\times\sin\theta_0=63\ (kN)$

Therefore, the total force acting on both ends of the tunnel invert were:

Shear force: $Q_0= Q_1+ Q_2=467\ (kN)$

Axial force: $N_0= N_1+ N_2=1296\ (kN)$

Bending moment: $M_0=591\ (kN\cdot m)$

### Settlement of tunnel bottom and pressure of rock mass when tunnel foundation was not strengthened

When the tunnel foundation was not reinforced, the tunnel foundation was soft soil, and the foundation stiffness was small, and the coefficient of foundation reaction was selected according to the empirical value [1], $K_1=8000\ kN/m^3$. The MATLAB program was compiled to solve the Equations 22 - 24, and the integral constant $C_0$, $C_1$ and $C_4$ in differential equations could be obtained.

$$C_0=12.39\times10^{-3},\ C_1=4.16\times10^{-3},\ C_4=7.95\times10^{-3}$$

We could get the Equation 26 - 29 when we substitute $C_0$, $C_1$ and $C_4$ into Equation 17 - 20:

$$y(\theta)=[12.39+4.16ch\alpha\theta cos\beta\theta+7.95sh\alpha\theta sin\beta\theta]\times10^{-3} \tag{26}$$

$$Q(\theta)=T[4.16(A_1sh\alpha\theta cos\beta\theta+A_2ch\alpha\theta sin\beta\theta)+7.95(A_1ch\alpha\theta sin\beta\theta-A_2sh\alpha\theta cos\beta\theta)]\times10^{-3} \tag{27}$$

$$N(\theta)=[-12.39Kr_1-4.16(2\alpha\beta Tsh\alpha\theta sin\beta\theta)+7.95(2\alpha\beta Tch\alpha\theta cos\beta\theta)]\times10^{-3} \tag{28}$$

$$M(\theta)=[12.39(Kr_1B_1+1)+4.16(2\alpha\beta)(B_1T-1)sh\alpha\theta sin\beta\theta-7.95(2\alpha\beta)(B_1T-1)ch\alpha\theta cos\beta\theta]\times10^{-3}/B_2 \tag{29}$$

From Equations 26 to 29, the deflection, shear force, axial force and bending moment of any section of the tunnel invert can be obtained.

### Internal force of tunnel invert

We calculated the internal force of any section of tunnel invert by Equation 27 - 29. Considering the symmetry of tunnel invert, half of the tunnel structure was calculated. The internal force diagrams were shown in Figure 3 - 5.

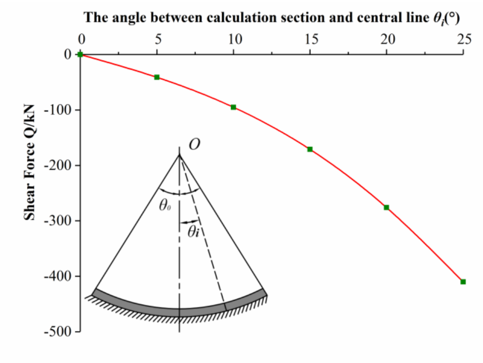


*Fig.3 - The shear force of tunnel invert (kN)*

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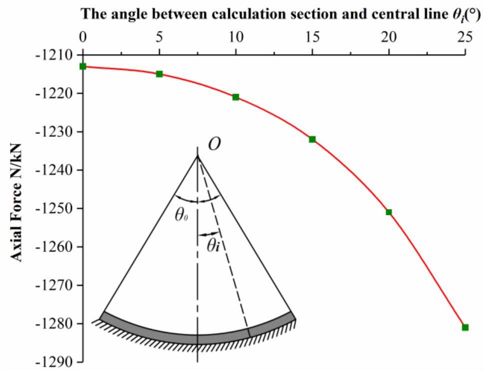


*Fig.4 - The axial force of tunnel invert (kN)*

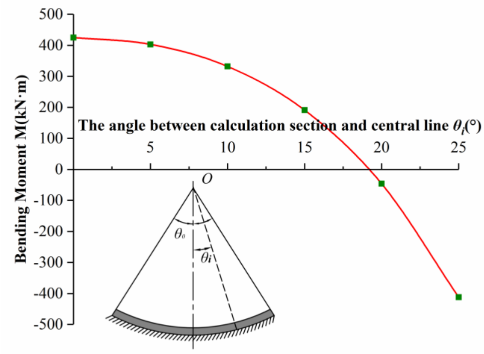


*Fig.5 - The bending moment of tunnel invert (kN· m)*

As shown in Figure 3 - 5, the centre and the waist of tunnel invert bore the tension stress, and the lateral bore compressive stress. However, the outer sides bore tension stress and the inner sides bore compressive stress at the corner of tunnel invert. The internal force at the corner of tunnel invert was greater, and there was stress concentration phenomenon.

The axial force of the tunnel invert was relatively large, and the maximum axial force was located at the corner of tunnel invert, which was $-1282kN$, and the direction was pointing to the inside of the tunnel. The shear force of the tunnel invert was relatively small, and the maximum shear force was located at the corner of tunnel invert, which was $-410kN$. The bending moment was relatively large, and the positive moment $425kN \cdot m$ was located at the centre of the tunnel invert. The force form was not conducive to the stability of the tunnel invert, and the inside of the tunnel invert was easy to be pulled to crack.

### Settlement of tunnel bottom

The radial displacement of any section of tunnel invert could be calculated by Equation 26, the vertical displacement of different sections of tunnel invert could be obtained by formula $S_i = y_i \cos\theta_i$, the calculated results are shown in Table 3.

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*Tab.3 - Settlement of tunnel bottom (mm)*

| The angle between the calculation section and central line $\theta_i$(°) | Horizontal distance between calculation section and tunnel invert center L（m） | Radial displacement $y$（mm） | Settlement of tunnel bottom $S$（mm） |
|---|---|---|---|
| 0 | 0 | 16.6 | 16.6 |
| 5 | 1.1 | 17.3 | 17.2 |
| 10 | 2.1 | 19.4 | 19.1 |
| 15 | 3.2 | 22.5 | 21.8 |
| 20 | 4.2 | 26.2 | 24.6 |
| 25 | 5.2 | 29.4 | 26.7 |

As shown in Table 3, the settlement of tunnel bottom was generally large when the tunnel foundation of soft loess was not reinforced. The settlement reached *26.7mm* at the corner of the tunnel bottom, and the settlement at the centre of the tunnel bottom reached *16.6mm*. The maximum differential settlement within the cross-section of tunnel bottom was as follow:

$$\Delta S_{max} = 26.7 - 16.6 = 10.1(\text{mm})$$

The horizontal distance between the centre and corner point of tunnel bottom was *5.2m*, therefore, the differential settlement of unit length was as follows:

$$\Delta S_{max}' = \Delta S_{max} / L = 1.94(\text{mm/m})$$

There is a big difference between the centre and corner point of tunnel bottom, so the tunnel invert was easy to crack. According to the distribution of the bending moment of the tunnel invert, the risk of cracking at the centre of tunnel invert was the biggest.

**Pressure of rock mass of tunnel bottom**

After calculating the radial displacement y of the tunnel invert, the elastic resistance of the tunnel foundation to the tunnel invert could be obtained by the formula (1). The normal elastic resistance was approximately equal to the pressure of rock mass of the tunnel bottom, assuming that the frictional force was not taken into account. The calculation results of pressure of rock mass were shown in Figure 6.

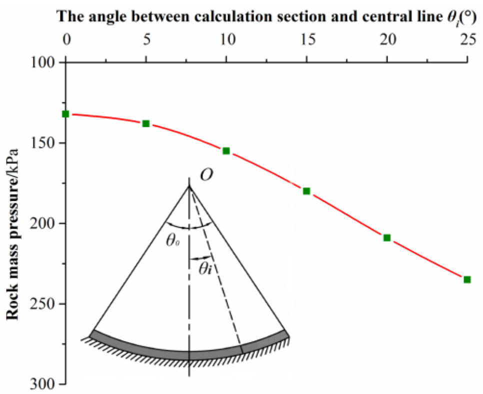


*Fig.6 - pressure of rock mass of tunnel bottom (kPa)*

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As shown in Figure 6, rock mass at the tunnel bottom was mainly subjected to compressive stress. The compressive stress at the centre of tunnel invert centre was the minimum, *132kPa*. The pressure of rock mass increased uniformly from the centre to the corner of tunnel invert, and the maximum compressive stress was reached *235kPa* at the corner of tunnel invert. The stress concentration was easy to occur due to the structural reasons at the corner of tunnel invert. The weight of secondary lining and the pressure of rock mass acted at the corner of tunnel invert, and the corner of tunnel invert bore the large load. There should be smooth transition between side wall and the corner of tunnel invert, and the section size at the corner of tunnel invert could be increased appropriately to improve the force situation in the process of tunnel construction.

## Settlement and pressure of rock mass of foundation reinforcement by high pressure jet grouting pile

When the tunnel foundation was reinforced by high pressure jet grouting pile, the bearing capacity and the stiffness of the tunnel foundation was greatly improved. The coefficient of foundation reaction was determined by field loading test, $K_2=60000\ kN/m^3$. The MATLAB program was compiled to solve the Equations 22 - 24, and the integral constant $C_0$, $C_1$ and $C_4$ in differential equations could be obtained.

$$C_0=16.6\times10^{-4},\quad C_1= -1.4\times10^{-4},\quad C_4=6.8\times10^{-4}$$

We can get the Equation 30 - 33 when substitute $C_0$, $C_1$ and $C_4$ into Equation 17 - 20:

$$y(\theta)=[16.6-1.4ch\alpha\theta cos\beta\theta+6.8sh\alpha\theta sin\beta\theta]\times10^{-4} \tag{30}$$

$$Q(\theta)=T[-1.4(A_1sh\alpha\theta cos\beta\theta+A_2ch\alpha\theta sin\beta\theta)+6.8(A_1ch\alpha\theta sin\beta\theta-A_2sh\alpha\theta cos\beta\theta)]\times10^{-4} \tag{31}$$

$$N(\theta)=[-16.6Kr_1+1.4(2\alpha\beta Tsh\alpha\theta sin\beta\theta)+6.8(2\alpha\beta Tch\alpha\theta cos\beta\theta)]\times10^{-4} \tag{32}$$

$$M(\theta)=[16.6(Kr_1B_1+1)-1.4(2\alpha\beta)(B_1T-1)sh\alpha\theta sin\beta\theta-6.8(2\alpha\beta)(B_1T-1)ch\alpha\theta cos\beta\theta]\times10^{-4}/B_2 \tag{33}$$

From Equations 30 - 33, the deflection, shear force, axial force and bending moment of any section of the tunnel invert could be obtained.

### Internal force of tunnel invert

We calculated the internal force of any section of tunnel invert by Equation 31 - 33. Considering the symmetry of tunnel invert, half of the tunnel structure was calculated. The internal force diagram was shown in Figure 7 - 9.

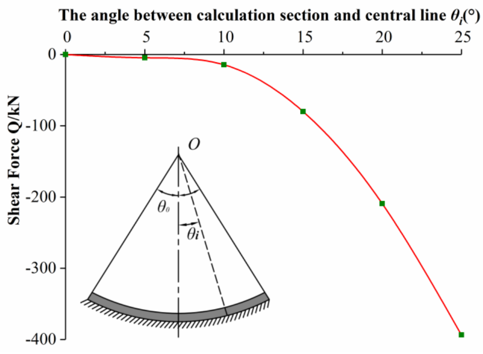


*Fig.7 - The shear force of tunnel invert (kN)*

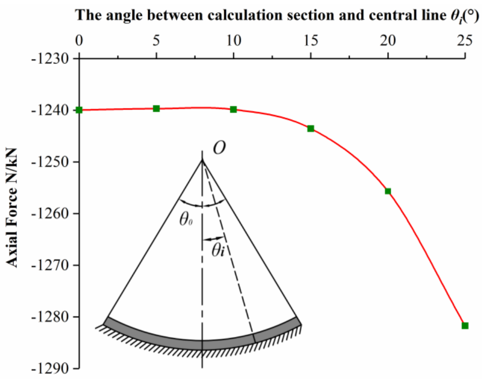


*Fig.8 - The axial force of tunnel invert (kN)*

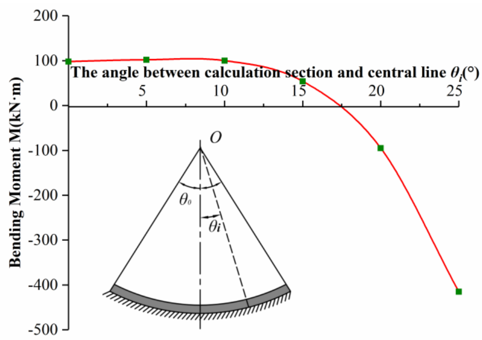


*Fig. 9 The bending moment of tunnel invert (kN.m)*

As shown in Figure 7 - 9, after the reinforcement of the tunnel foundation, the internal force of the tunnel invert was similar to that without reinforcement. The inside at the centre and the waist of tunnel bottom bore the tension stress, and the lateral bore compressive stress. The outer side at the corner of tunnel invert bore tension stress, and the inner side bore compressive stress. However, the stress condition of the tunnel invert had been improved. The axial force at the centre the tunnel invert increased slightly and the shear force and bending moment decreased. The maximum positive bending moment at the centre of the tunnel invert decreased to $98kN \cdot m$, which greatly improved the stress state of the tunnel invert.

**Settlement of tunnel bottom**

The radial displacement of any section of tunnel invert could be calculated by Equation 30, the vertical displacement of different sections of tunnel invert could be obtained by formula $S_i = y_i \cos\theta_i$,

the calculation results were shown in Table 4.

*Tab.- 4 Settlement of tunnel bottom (mm)*

| The angle between the calculation section and central line $\theta_i$(°) | Horizontal distance between calculation section and tunnel invert center L（m） | Radial displacement y（mm） | Settlement of tunnel invert S（mm） |
|---|---|---|---|
| 0 | 0 | 1.5 | 1.5 |
| 5 | 1.1 | 1.7 | 1.7 |
| 10 | 2.1 | 2.2 | 2.2 |
| 15 | 3.2 | 3.1 | 3.0 |
| 20 | 4.2 | 4.1 | 3.9 |
| 25 | 5.2 | 4.7 | 4.3 |

As shown in Table 3 and Table 4, the settlement of the tunnel bottom was greatly reduced when the tunnel foundation was reinforced by jet grouting pile. The settlement at the corner of the tunnel invert decreased to *4.3mm*, and the differential settlement of unit length was as follows:

$$\Delta S_{max}' = \Delta S_{max} / L = 0.54 (mm/m)$$

The settlement of the tunnel invert was smaller and the overall settlement was more harmonious, which met the requirements of the standard. Thus, the bearing capacity and stiffness of tunnel foundation was greatly improved after the jet grouting pile was used to reinforce the tunnel foundation, and the reinforcement effect was good.

### Pressure of rock mass of tunnel bottom

The calculation results of pressure of rock mass were shown in Figure 10.

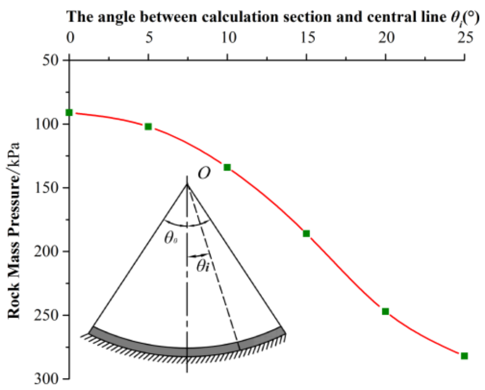


*Fig.10 - pressure of rock mass of tunnel bottom (kPa)*

As shown in Figure 10, the pressure of rock mass at tunnel bottom was similar to that without reinforcement, the compressive stress at the centre of tunnel invert was the smallest, and the maximum compressive stress at the corner of tunnel invert, after foundation reinforcement, the compressive stress at the corner of tunnel invert increased more obviously, and increased from *235kPa* to *282kPa*. The stress concentration phenomenon at the corner of tunnel invert was more obvious, which was related to the substantial increase of the bearing capacity of the tunnel foundation after the foundation reinforcement.

## CONCLUSIONS

(1) When the tunnel foundation was not strengthened, the inside at the centre and the waist of tunnel invert bore the tension stress, and the lateral bore compressive stress; the outside at the foot of the tunnel invert bore tension stress and the inside bore compressive stress; the internal force at the foot of tunnel invert was greater, the axial force and bending moment of the tunnel invert were relatively large, and the greater positive moment was inside the centre of the tunnel invert. The maximum positive bending moment at the centre of the tunnel invert decreased greatly after the reinforcement of the tunnel foundation.

(2) When the tunnel foundation was not strengthened, the settlement of tunnel bottom was generally large; the settlement at the corner of tunnel invert reached *26.7mm*, and the maximum differential settlement within the range of tunnel invert reached *10.1mm*; the risk of tension crack at the centre of tunnel invert was larger. After tunnel foundation was strengthened, the settlement of tunnel bottom decreased greatly, and the overall settlement was more coordinated, and the reinforcement effect was good.

(3) When the tunnel foundation was not strengthened, theoretical calculation results showed that the rock mass at tunnel bottom was mainly subjected to compressive stress; the compressive stress at the centre of the tunnel invert was minimum, and it was large at the corner of tunnel invert. After the tunnel foundation was strengthened, the pressure of rock mass at tunnel bottom was similar to that without reinforcement; the compressive stress at the corner of tunnel invert increased obviously and the stress concentration phenomenon was more obvious.

## COMPETING INTERESTS

The authors declare that there is no conflict of interests regarding the publication of this paper.

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