GEOMETRY CONSTRUCTION METHOD OF HEX-TRI RECIPROCAL FRAME

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ABSTRACT

In this paper the geometrical characteristics and construction method of hex-tri reciprocal frame are studied. The naming rules of structural units, members and joints of hex-tri reciprocal frame are proposed. Based on the study of relationships between the structural geometrical parameters, the formulas for the joint coordinates of the structural unit are derived, and the calculating method for normalized direction vectors of the unit members is also developed. The geometry construction method of hex-tri reciprocal frame is established in this paper. By this method the whole structure is formed as an assembly of the units arranged in rings about the structural center. The influences of the diameter, the length and the binding length of the member on the rise of hex-tri reciprocal frame are analyzed by an example. If the rise of hex-tri reciprocal frame needs to be specified in the design, the values of the diameter, the length and the binding length of the structural member need to be coordinated to meet the design requirements.

KEYWORDS

Hex-tri reciprocal frame, Structural geometry, Geometrical characteristics, Structural unit, Construction method

INTRODUCTION

Reciprocal frame is a kind of self-balancing structure. The model is arranged in such a way as to form a closed circuit of mutually supporting elements [1]. The support points are different, and not all of them are located at the end points of the member. The windmill-liked roof shown in Figure 1 is a simple example of reciprocal frame.

Currently there is a clear trend towards prefabricated construction in building industry, and there is much application potential for reciprocal frame as an applicable structure type for this tendency. Compared with the traditional large-span space structure, joints of reciprocal frame are simple and unified. Generally, there is a large number of members and joints in traditional space structures, and many members are connected by the same joint in some cases. So the joints are complex in construction, and the assembly errors are likely to be caused. The reciprocal frame joint usually connects only two members, which avoids the possible multi-member intersection in traditional space structures [2,3]. Generally, there is only one type of joint with the uniform dimension in reciprocal frame. This makes it easy for mass manufacturing while avoiding drawbacks of the traditional space structures [4].

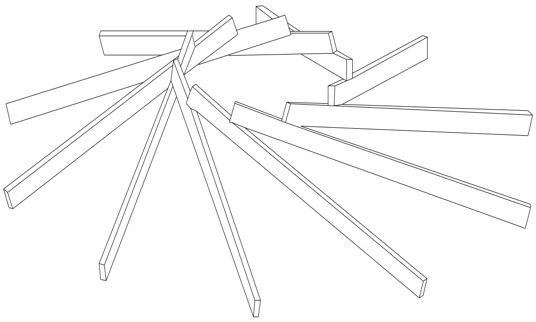

Fig. 1 – Windmill-liked reciprocal frame

Although the early reciprocal frame emerged centuries ago, systematic research on this structure began in the 1990s. Graham Brown firstly proposed the concept of reciprocal frame [2]. Chilton and Choo carried out a series of studies on geometry, mechanical properties and structural construction of reciprocal frames [5,6]. Popovic published a monograph with regard to reciprocal frame [7]. While teaching in Hong Kong, Bertin developed a variety of reciprocal frame models based on the reciprocal principle [8]. Baverel proposed a shape forming method for reciprocal frame based on genetic algorithm and gradient algorithm [9]. Rizzuto studied the geometrical configuration of reciprocal frames using space analytic geometry method [10]. Parigi and Peng et al. proposed some methods to generate new types of reciprocal structure [11-13]. Rizzuto and Olga studied connection systems in reciprocal frames and mutually supported elements space structure [14]. Dario et al. analysed the static and dynamic properties of planar reciprocal components [15]. Some architects carried out some engineering practice based on the concept of reciprocal structure. For example, Brown built a Round House which is a reciprocal frame using metal material [7]. The external maintenance structure has been constructed at Mount Rokko-Shidare Observatory in Japan by using the reciprocal space structure [16]. Using aluminium alloy as the material, Gelez built a reciprocal frame in Bibracte, France, as the exhibition hall of cultural relics [17].

At present, systematic calculation and design method for reciprocal frame have not been established. So the reciprocal frame has not been used at a large scale. The completed structures of reciprocal frames are usually small in scale. Different from the traditional space structure, members of reciprocal structures are eccentrically connected. This leads to a quite different geometric construction of reciprocal structures from the traditional structure. It is necessary to propose the geometric construction method of reciprocal frame, which is the basic requirement for further analysis of structure mechanical behaviour. In this paper, the formulas for the joint coordinates of the structural units which compose the hex-tri reciprocal frame are derived, and the calculating method for normalized direction vectors of the unit members is also developed. The naming rules of structural units, members and joints of hex-tri reciprocal frame are proposed. The geometry construction method of hex-tri reciprocal frame is established in this paper by which the whole structure is formed as an assembly of the units arranged in rings about the structural center.

STRUCTURAL UNIT

The hex-tri reciprocal frame is shown in Figure 2. It is constituted of hexagonal and trilateral grids which are alternately distributed.

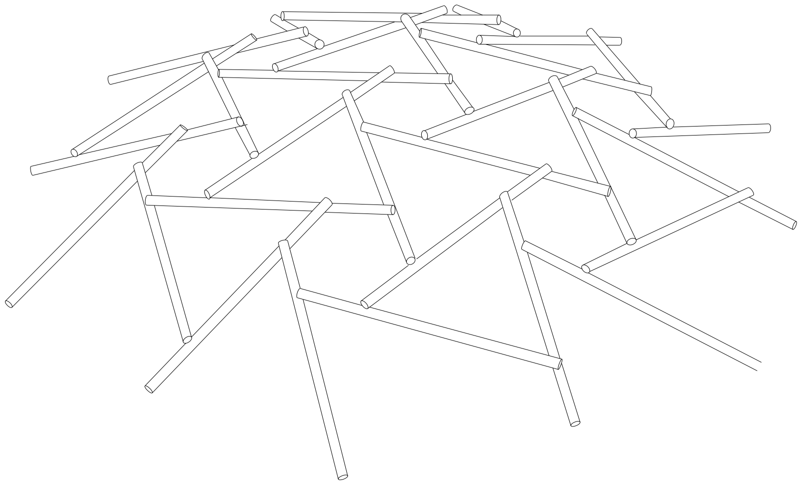

Fig. 2 – The hex-tri reciprocal frame

The hex-tri reciprocal frame is constituted of continuously arranged units which are shown in Figure 3. The unit consists of six members, and each member joint the adjacent one at an angle of 60°. Axes of the structure reciprocal members will never meet, so there is a space between the adjacent axes. This would cause the eccentricity. Members of the unit are numbered counter clockwise as H_{01}^{1}, H_{01}^{2}, H_{01}^{3}, H_{01}^{4}, H_{01}^{5} and H_{01}^{6}. Direction vectors of each member are numbered as $\overrightarrow{h_{01}^{1}}$, $\overrightarrow{h_{01}^{2}}$, $\overrightarrow{h_{01}^{3}}$, $\overrightarrow{h_{01}^{4}}$, $\overrightarrow{h_{01}^{5}}$, $\overrightarrow{h_{01}^{6}}$. When any geometrical parameter of a member equals to that of any other members of the unit, the six outside endpoints of the unit are in the same plane, so are the six inside endpoints and the six joints. As shown in Figure 3, a 3D rectangular coordinate system with XOY plane parallel to those of the six outside endpoints as well as the six inside endpoints of the unit is established. The reciprocal members overlap each other, so the axes are not intersected and the contact point does not locate on the axis. In this paper, the contact point is called reciprocal point and the corresponding points on its upper and lower axis are called axis joint. In the unit, axis joints of the supported members are N_{01}^{1}, N_{01}^{2}, N_{01}^{3}, N_{01}^{4}, N_{01}^{5}, and N_{01}^{6} respectively. Axis joints of the supporting members are $\overline{N}_{01}^{1}$, $\overline{N}_{01}^{2}$, $\overline{N}_{01}^{3}$, $\overline{N}_{01}^{4}$, $\overline{N}_{01}^{5}$ and $\overline{N}_{01}^{6}$ respectively.

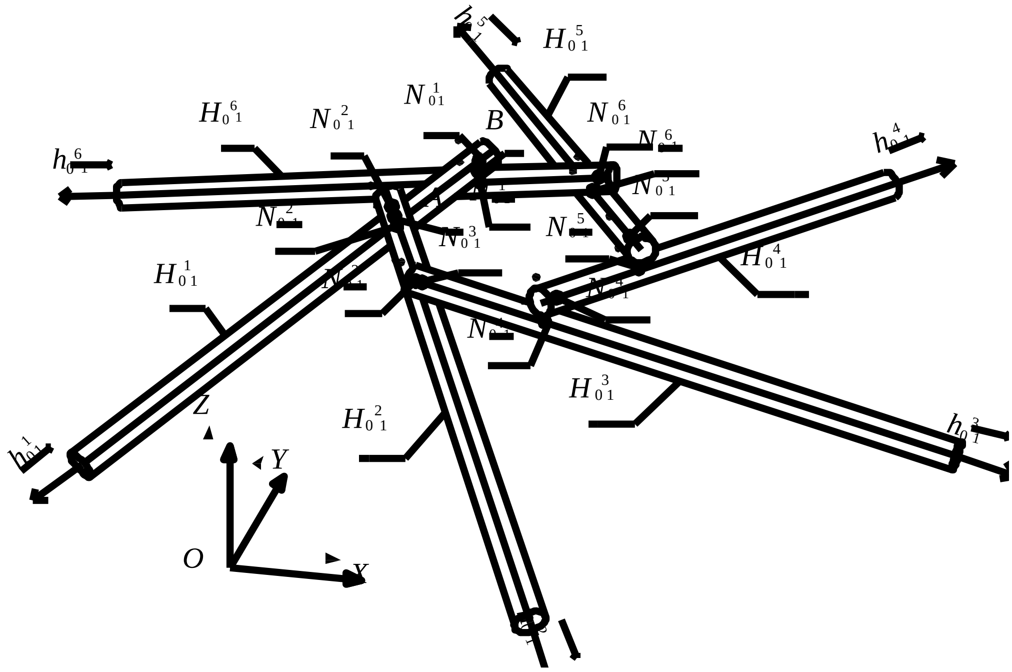

Fig. 3 – Numbers of the unit members and joints

The longitudinal section of member H_{01}^1 is shown in Figure 4. In Figure 4, L is the member length; L_e is the binding length; d is the diameter of the cross section; e is the distance between the two axes of the reciprocal members; s is the distance between the two axis joints; θ is the angle between the member and the horizontal plane;

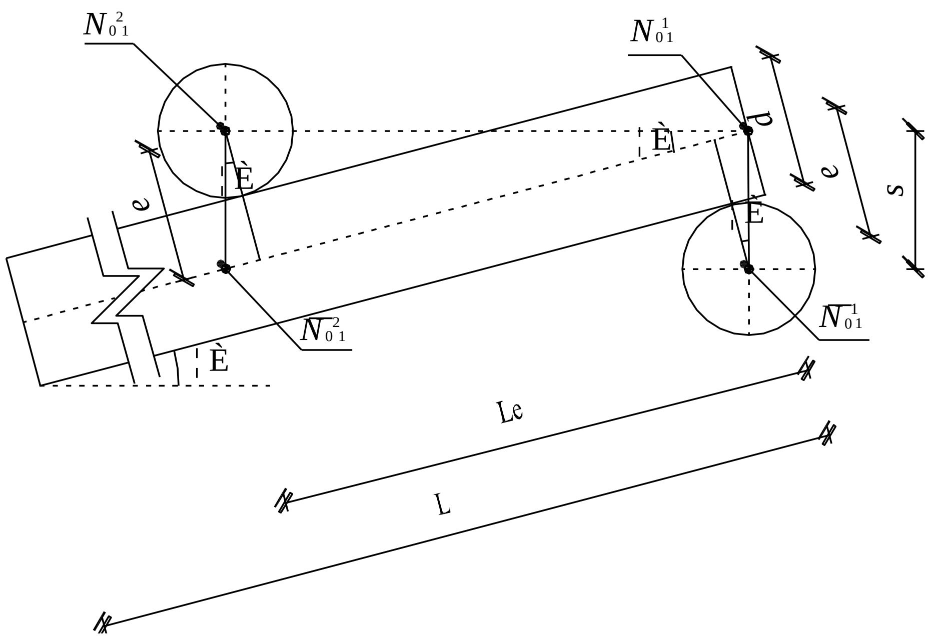

Fig. 4 – Longitudinal section of the member

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As shown in Figure 4, when the structural member has the same cross section diameter,

$$e = d \tag{1}$$

Then the distance s between the two axis joints is given as follows:

$$s = \frac{e}{\cos\theta} = \frac{d}{\cos\theta} \tag{2}$$

Based on Figure 4, the relationship of L_e, θ and s can be derived as follows:

$$L_e \sin\theta = s \tag{3}$$

The following equation can be obtained by plugging Eq. (2) into Eq. (3).

$$L_e \sin\theta = \frac{d}{\cos\theta} \tag{4}$$

Let the coordinate of N_{01}^1 be (x_1, y_1, z_1). The distance between N_{01}^1 and $\overline{N}_{01}^1$ can be obtained as $d/\cos\theta$ based on Eq. (2). So the coordinate of $\overline{N}_{01}^1$ can be given as follows:

$$\overline{N}_{01}^1 \left(x_1, y_1, z_1 - \frac{d}{\cos\theta} \right) \tag{5}$$

The coordinate of $\overline{N}_{01}^1$ can be obtained by plugging Eq. (4) into Eq. (5).

$$\overline{N}_{01}^1 \left(x_1, y_1, z_1 - L_e \sin\theta \right) \tag{6}$$

Let the coordinate of N_{01}^2 be (x_2, y_2, z_2). The formula of this coordinate can be derived based on Figure 3.

$$x_2 = x_1 - \frac{L_e \cos\theta}{2} \tag{7a}$$

$$y_2 = y_1 - \frac{\sqrt{3} L_e}{2} \tag{7b}$$

$$z_2 = z_1 \tag{7c}$$

So the coordinate of N_{01}^2 is

$$N_{01}^2 \left(x_1 - \frac{L_e \cos\theta}{2}, y_1 - \frac{\sqrt{3} L_e \cos\theta}{2}, z_1 \right) \tag{8}$$

Likewise, the coordinate of $\overline{N}_{01}^2$ can be given as follows:

$$\overline{N}_{01}^2 \left(x_1 - \frac{L_e \cos\theta}{2}, y_1 - \frac{\sqrt{3} L_e \cos\theta}{2}, z_1 - L_e \sin\theta \right) \tag{9}$$

Similarly, coordinates of each axis joint in the unit can be derived as follows:

$$N_{01}^{3}\left(x_1, y_1-\sqrt{3}L_e\cos\theta, z_1\right) \tag{10a}$$

$$\overline{N}_{01}^{3}\left(x_1, y_1-\sqrt{3}L_e\cos\theta, z_1-L_e\sin\theta\right) \tag{10b}$$

$$N_{01}^{4}\left(x_1+L_e\cos\theta, y_1-\sqrt{3}L_e\cos\theta, z_1\right) \tag{10c}$$

$$\overline{N}_{01}^{4}\left(x_1+L_e\cos\theta, y_1-\sqrt{3}L_e\cos\theta, z_1-L_e\sin\theta\right) \tag{10d}$$

$$N_{01}^{5}\left(x_1-\frac{3L_e\cos\theta}{2}, y_1-\frac{\sqrt{3}L_e\cos\theta}{2}, z_1\right) \tag{10e}$$

$$\overline{N}_{01}^{5}\left(x_1-\frac{3L_e\cos\theta}{2}, y_1-\frac{\sqrt{3}L_e\cos\theta}{2}, z_1-L_e\sin\theta\right) \tag{10f}$$

$$N_{01}^{6}\left(x_1+L_e\cos\theta, y_1, z_1\right) \tag{10g}$$

$$\overline{N}_{01}^{6}\left(x_1+L_e\cos\theta, y_1, z_1-L_e\sin\theta\right) \tag{10h}$$

Based on the axis joint coordinates, the normalized direction vectors of the members can be obtained as follows:

$$\overrightarrow{h_{01}^{1}}=\frac{\overrightarrow{O\overline{N}_{01}^{2}}-\overrightarrow{ON_{01}^{1}}}{L_e}=\left(-\frac{\cos\theta}{2},-\frac{\sqrt{3}\cos\theta}{2},-\sin\theta\right) \tag{11a}$$

$$\overrightarrow{h_{01}^{2}}=\frac{\overrightarrow{O\overline{N}_{01}^{3}}-\overrightarrow{ON_{01}^{2}}}{L_e}=\left(\frac{\cos\theta}{2},-\frac{\sqrt{3}\cos\theta}{2},-\sin\theta\right) \tag{11b}$$

$$\overrightarrow{h_{01}^{3}}=\frac{\overrightarrow{O\overline{N}_{01}^{4}}-\overrightarrow{ON_{01}^{3}}}{L_e}\left(\cos\theta,0,-\sin\theta\right) \tag{11c}$$

$$\overrightarrow{h_{01}^{4}}=\frac{\overrightarrow{O\overline{N}_{01}^{5}}-\overrightarrow{ON_{01}^{4}}}{L_e}=\left(\frac{\cos\theta}{2},\frac{\sqrt{3}\cos\theta}{2},-\sin\theta\right) \tag{11d}$$

$$\overrightarrow{h_{01}^{5}}=\frac{\overrightarrow{O\overline{N}_{01}^{6}}-\overrightarrow{ON_{01}^{5}}}{L_e}=\left(-\frac{\cos\theta}{2},\frac{\sqrt{3}\cos\theta}{2},-\sin\theta\right) \tag{11e}$$

$$\overrightarrow{h_{01}^{6}}=\frac{\overrightarrow{O\overline{N}_{01}^{1}}-\overrightarrow{ON_{01}^{6}}}{L_e}=\left(-\cos\theta,0,-\sin\theta\right) \tag{11f}$$

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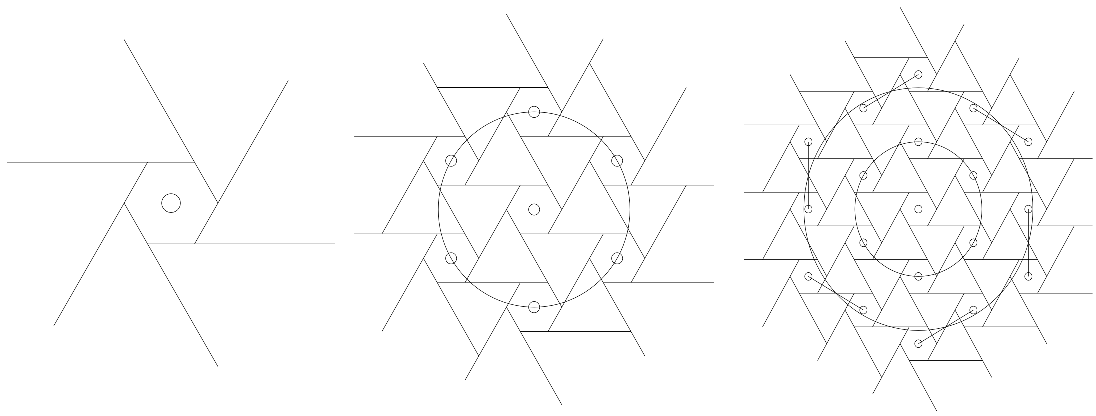
GEOMETRICAL CONSTRUCTION OF THE STRUCTURE

The whole structure can be seen as a unit assembly in a regular manner. The hex-tri reciprocal frame is formed by the hex-tri units arranged in rings about the structural center. The structure geometry method explicates an example structure consisting of elements arranged in two rings, as shown in Figure 5. There is a central unit which locates at the structure center (Figure 5a). The hex-tri reciprocal frame is constructed by arranging the units in the first ring (Figure 5b) and then in the second ring (Figure 5c).



(a) (b) (c)

Fig. 5 – An example hex-tri reciprocal structure

The numbers of the members in the central unit are as same as those illustrated in Figure 3. The structural member is represented by H_{ij}^{k}, where i indicates that the member locates in the i^{th} ring of the structure; j indicates that the member locates in the j^{th} unit; k indicates that this is the k^{th} member of the unit. The unit is represented by U_{ij}, where i indicates that the unit locates in the i^{th} ring of the structure; j indicates that this is the j^{th} unit in the ring. In the hex-tri reciprocal frame, one member is shared by two units arranged in two rings. If the 1st member of 1st unit in the i^{th} ring is shared by a unit in the i+1th ring, this unit is defined as the 1st unit in the i+1th ring. The other units are numbered counter clockwise. As shown in Figure 6, the unit in the 1st ring that shares H_{01}^{1} of the central unit is defined as U_{11}. The other units in the 1st ring are defined as U_{12}, U_{13}, U_{14}, U_{15} and U_{16} counter clockwise. The numbering order of members in each unit is as same as that of the central unit. The shared member has different number when it is used by different unit. Some numbers of unit, member and joint are shown in Figure 6.

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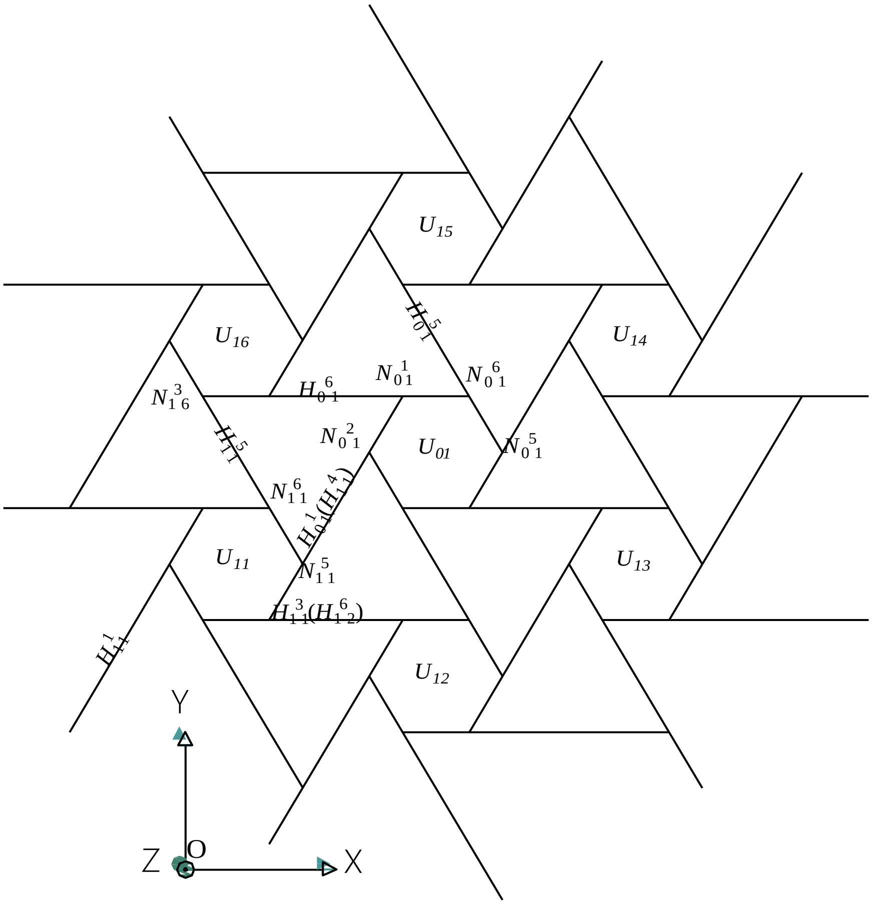


Fig. 6 – Some numbers of unit, member and joint

The coordinate of $N_{01}^{1}\left(x_1, y_1, z_1\right)$ needs to be determined firstly. Coordinates of the axis joints of the central unit can be obtained based on Eq. (6), Eq. (8), Eq. (9) and Eq. (10). Normalized direction vectors of the central unit members can be obtained based on Eq. (11).

Since N_{16}^{3} is on the member H_{01}^{6} where both $\overline{N}_{01}^{1}$ and N_{01}^{6} locate, its coordinate can be derived based on the coordinates of these two joints.

$$N_{16}^{3}\left(x_1-\left(L-L_e\right)\cos\theta, y_1, z_1-L\sin\theta\right) \tag{12}$$

Likewise, since $\overline{N}_{11}^{5}$ is on the member H_{01}^{1} where both N_{01}^{1} and $\overline{N}_{01}^{2}$ locate, its coordinate can be derived based on the coordinates of these two joints.

$$\overline{N}_{11}^{5}\left(x_1-\left(L-L_e\right)\cos\theta\cos 60^\circ, y_1-\left(L-L_e\right)\cos\theta\sin 60^\circ, z_1-\left(L-L_e\right)\sin\theta\right) \tag{13}$$

Based on Eq.(3) and Eq.(12), the coordinate of axis joint $\overline{N}_{16}^{3}$ can be obtained as follows:

$$\overline{N}_{16}^{3}\left(x_1-\left(L-L_e\right)\cos\theta, y_1, z_1-L\sin\theta-L_e\sin\theta\right) \tag{14}$$

Based on Eq.(3) and Eq.(13), the coordinate of axis joint N_{11}^{5} can be obtained as follows:

$$N_{11}^{5}\left(x_1-\frac{(L-L_e)\cos\theta}{2},y_1-\frac{\sqrt{3}(L-L_e)\cos\theta}{2},z_1-L\sin\theta+2L_e\sin\theta\right) \tag{15}$$

The direction vector of member H_{11}^{5} can be derived based on Eq. (14) and Eq. (15):

$$\overrightarrow{h_{11}^{5}}=\frac{\overrightarrow{ON_{16}^{3}}-\overrightarrow{ON_{11}^{5}}}{L-L_e}=\left(-\frac{\cos\theta}{2},\frac{\sqrt{3}\cos\theta}{2},-\frac{3L_e\sin\theta}{L-L_e}\right) \tag{16}$$

$$\text{Let}\quad \overrightarrow{h_{11}^{5}}=\overrightarrow{h_{01}^{5}}[T] \tag{17}$$

When Eq.(11e) and Eq. (16) is plugged into Eq. (17), the following Equation is obtained:

$$[T]=\begin{bmatrix}1 & 0 & 0\\ 0 & 1 & 0\\ 0 & 0 & \frac{3L_e}{L-L_e}\end{bmatrix} \tag{18}$$

Structural unit U_{12} ~U_{16} is constructed in a manner similar to that of U_{11}. H_{1j}^{m} of unit U_{1j} in the first ring and H_{01}^{j} of the central unit U_{01} are one and the same member. The relationship between the direction vectors $\overrightarrow{h_{1j}^{n}}$ and $\overrightarrow{h_{01}^{n}}$ is given as follows:

$$\overrightarrow{h_{1\,j}^{n}}=\overrightarrow{h_{01}^{n}}\begin{bmatrix}1 & 0 & 0\\ 0 & 1 & 0\\ 0 & 0 & \frac{3L_e}{L-L_e}\end{bmatrix} \tag{19}$$

Where m and n are integers range from 1 to 6, and $|m-j|=3$; $|n-j|=4$. Based on the constitution method of unit U_{01} in the center and unit U_{11} in the first ring, the rest units are formed, and so on the whole structure can be formed then. The modelling process of hex-tri reciprocal frame is illustrated in Figure 7.

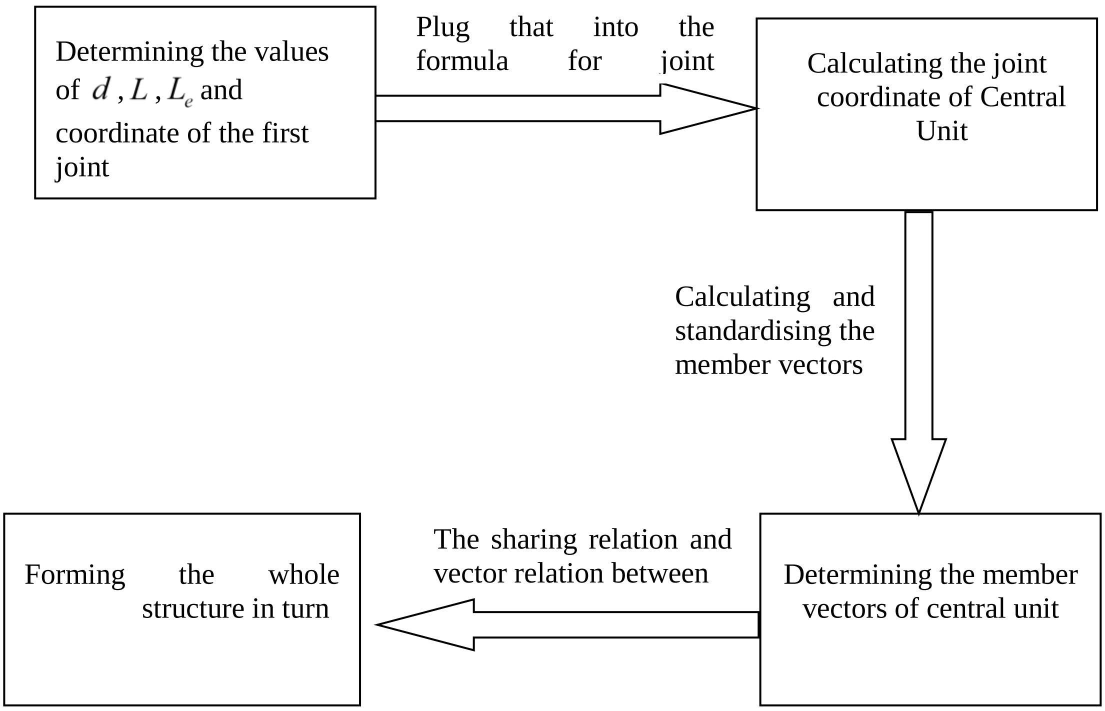


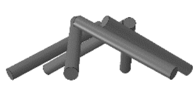
Fig. 7 – Modelling process of hex-tri reciprocal frame

EXAMPLE

When the values of the ring number n, the member cross section diameter d, the member length L and the binding length L_e are specified, the hex- tri reciprocal frame is constructed based on the structural geometry construction method proposed in this paper. The rise of hex-tri reciprocal frame varies with the change of the values of d, L and L_e. The influences of the geometrical parameters d, L, and L_e on the structural rise are examined based on a hex-tri frame composed of the central unit.

The influence of d on the structural rise *h* is examined by changing the value of d while fixing the values of L and L_e. The result is shown in Table 1.

Tab. 1 - The influence of d on the structural rise h

No.	1	2	3	4	5
L/m	4.00	4.00	4.00	4.00	4.00
L_e /m	1.00	1.00	1.00	1.00	1.00
d/mm	50.00	150.00	250.00	350.00	450.00
h/mm	200.00	600.00	1000.00	1400.00	1800.00
Structure sketch					

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It can be seen from Table 1 that the structural rise gets bigger linearly with the increment of the member cross section diameter *d*. The influence of L on the structural rise *h* is examined by changing the value of L while fixing the values of d and L_e. The result is shown in Table 2.

Tab. 2 - The influence of L on the structural rise h

No.	1	2	3	4	5
d/mm	250.00	250.00	250.00	250.00	250.00
L_e /m	1.00	1.00	1.00	1.00	1.00
L/m	3.00	3.50	4.00	4.50	5.00
h/mm	750.00	875.00	1000.00	1125.00	1250.00
Structure sketch					

It can be seen from Table 2 that the structural rise gets bigger linearly with the increment of the member cross section diameter *L*. The influence of L_e on the structural rise *h* is examined by changing the value of L_e while fixing the values of d and L. The result is shown in Table 3.

Tab. 3 - The influence of L_e on the structural rise h

No.	1	2	3	4	5
d/mm	250.00	250.00	250.00	250.00	250.00
L/m	4.00	4.00	4.00	4.00	4.00
L_e /m	0.50	0.75	1.00	1.25	1.50
h/mm	666.67	800.00	1000.00	1333.33	2000.00
Structure sketch					

It can be seen from Table 3 that the structural rise gets smaller with the increment of the member cross section diameter *L*.

In the design of the traditional space structure, the rise can be specified independently. Differing from the traditional space structure, the rise of the hex-tri reciprocal frame depends on d, L and L_e. If the rise of hex-tri reciprocal frame needs to be specified in the design, the values of the diameter, the length and the binding length of the structural member need to be coordinated to meet the design requirements.

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CONCLUSIONS

(1) The geometrical characteristics and construction method of hex-tri reciprocal frame are studied. The naming rules of structural units, members and joints of hex-tri reciprocal frame are proposed.

(2) Based on the study of relationships between the structural geometrical parameters, the formulas for the joint coordinates of the structural unit are derived, and the calculating method for normalized direction vectors of the unit members is also developed.

(3) The geometry construction method of hex-tri reciprocal frame is established in this paper. By this method the whole structure is formed as an assembly of the units arranged in rings about the structural center.

(4) The influences of the diameter, the length and the binding length of the member on the rise of hex-tri reciprocal frame are analyzed by an example. If the rise of hex-tri reciprocal frame needs to be specified in the design, the values of the diameter, the length and the binding length of the structural member need to be coordinated to meet the design requirements.

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